

Radial Migration in Spiral Galaxies

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Abstract

The redistribution of stars in galactic disks is an important aspect of disk galaxy evolution. Stars that efficiently migrate in such a way that does not also appreciably heat their orbits can drastically affect the stellar populations observed today and therefore influence constraints derived from such observations. Unfortunately, while the theoretical understanding of the migration process is becoming increasingly robust, there are currently few specific observable predictions. As a result, we do not yet have a clear handle on whether the process has been important for the Milky Way in the past or how to constrain it. I discuss some of the expected qualitative outcomes of migration as well as some current controversies.

1 Introduction

Galactic archaeology attempts to infer past history of our Milky Way disk from the rich datasets detailing stellar populations in terms of their chemistry, kinematics and ages. However, in order to ease interpretation, “population” is often loosely used to imply membership based on *current* cohabitation of a similar part of the Galaxy. This is borne out in the use of terminology like “the Solar Neighborhood” and the characterization of galactic disk properties in terms of the one-dimensional distance from the galactic center. For the most part, such groupings seem to make sense. After all, if the guiding radius R_g is defined by a star’s angular momentum, i.e. $R_g \sim L_z/V_c$ where L_z is the z component of the angular momentum and V_c is the circular velocity, then the deviations from this radius during its orbit due to excess orbital energy are constrained to < 2 kpc *for the oldest stars* near the sun (Binney, 2007). Furthermore assuming that the disk is mostly axisymmetric, then viewing the Solar Neighborhood as a representative random sample around $\sim R_\odot$ and using it to infer past history at this radius seems reasonable.

This simple assumptions is remarkably powerful. By combining our knowledge of stellar evolution and metal production, just the metallicity distribution function (MDF) and the age-metallicity relation (AMR) are needed to infer the entire history of (this part of) the MW disk. Unfortunately, in its simplest incarnation, this type of modeling fails spectacularly in what is largely known as the ‘G-dwarf problem’, because it over-predicts the relative number of metal-poor stars (e.g. Searle & Sargent 1972; Tinsley 1975). One of the explanations for too many metal-poor stars is that the available gas is depleted too soon and therefore the simplest solution is to allow for an inflow of gas from the outside (Larson, 1974; Lynden-Bell, 1975). In this way, it is recognized that of course the Solar Neighborhood does not exist in isolation from other parts of the Galaxy, but that it is only a part of the whole.

Although gas is regarded free to stream in and out of the volume defining a local patch of the disk, stars are not usually assumed to do so due to small radial orbital excursions described

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above. While the authors themselves cautioned about observational bias resulting in the AMR, Edvardsson et al. (1993) data has frequently been used to support the idea that local stars of different ages represented generations of stars born out of the same recycled gas. This fits well with most one-dimensional chemical evolution models (e.g. Prantzos & Aubert 1995), which robustly predict a strong age-metallicity relation. However, more recent data (e.g. the Geneva-Copenhagen survey Holmberg et al. 2009 and its various re-interpretations, and the Gaia-ESO survey Bergemann et al. 2014) show instead that the AMR is flat, that is, the AMR does not exist (Bergemann et al. 2014 do note, however, the lack of solar-metallicity stars older than 10 Gyr). The most striking feature of these observations is the large scatter in $[\text{Fe}/\text{H}]$ at each age, which cannot easily be explained away by asymmetric drift arguments. These observations therefore expose a need to rethink disk evolution and let the stars jump out of the box.

2 Stellar Migration

If stars from very different parts of the disk can be found together today, and if there is a metallicity gradient in the star-forming gas, then a large spread in metallicity at each stellar age is expected. Consequently, much like gas flows were the answer to the G-dwarf problem, radial mixing of stars presents a convenient solution for the dispersed AMR problem described above. The idea that stars should be diffusing through the disk was first proposed by Wielen (1977), who suggested that determining the birth places of stars may be complicated due to orbital diffusion. However, because only diffusion due to random scattering off irregularities like GMCs was considered, the guiding radii R_g were shown to stay nearly constant (Grenon, 1987). Nevertheless, Wielen et al. (1996) postulated that the Sun itself must have moved away from its birthplace in the inner disk, based on the fact that its metallicity is high compared to other stars of similar age in its vicinity. Using simplified arguments about the evolution of the Milky Way metallicity gradient, they constrained the Sun's birthplace to be ~ 2 kpc closer to the Galactic center.

However, Sellwood & Binney (2002) showed that the treatment used in Wielen (1977), which considered only random scattering off GMCs, was insufficient as it did not take into account resonant interactions with spiral arms. In particular, Sellwood & Binney (2002) showed that if a star's orbital speed matches the speed of a passing transient spiral wave, i.e. the star is at the corotation resonance of the spiral, the star's angular momentum L_z can be altered on a very short timescale. Due to the conservation of the Jacobi integral in such an interaction, the changes in energy and angular momentum are related simply by

$$\Delta E = \Delta L_z \Omega_p, \quad (1)$$

where Ω_p is the pattern speed of the passing spiral wave. Sellwood & Binney (2002) showed also that the radial action, J_R of the migrating star changes as

$$\Delta J_R = \frac{\Omega_p - \Omega}{\omega_R} \Delta L_z, \quad (2)$$

where Ω and ω_R are the azimuthal and radial frequencies of the star. Therefore, at corotation, angular momentum and energy are exchanged without causing additional heating to the orbit, in stark contrast to changes in L_z at the Lindblad resonances where stars heat very efficiently. This remarkable result presents a serious hurdle for efforts trying to reconstruct Galactic history from present-day observations. Stellar kinematics are typically used to try and infer the dynamical histories of stars, i.e. postulate whether they had heated, were accreted, had interacted with the bar etc. If stars may change their guiding radii by *several* kpc without acquiring any kinematical trace of the process, it becomes very difficult to disentangle which stars are actually telling the story of this part of the Galaxy, and which are part of another history altogether.

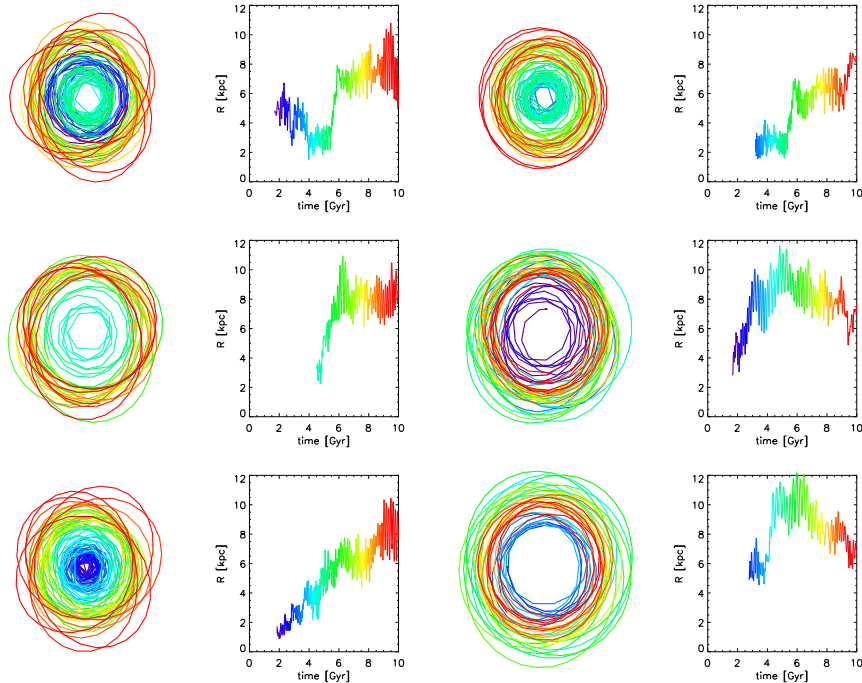


Figure 1: Sample orbits from an idealized disk-formation simulation (Roškar et al., 2012). All particles are selected to be between 7-9 kpc at the end of the simulation.

There is an important caveat with regard to the corotation resonance. If the perturbing pattern is steady, the corotation resonance results simply in orbit trapping. In this case, a star on the inside of corotation gains angular momentum passing to the outside of corotation, where it is now leading the pattern slightly and being pulled back by the overdensity loses the previously gained angular momentum. This trapping results in a horseshoe orbit which in the steady-state remains at a constant L_z . Therefore, for the corotation resonance to impart secular changes to a star's angular momentum, the pattern must be transient but last just long enough to deposit the star on the other side of the resonance before it gets pulled back in the other direction.

3 Stellar migration mechanisms in simulations

The expectation from Sellwood & Binney (2002) is therefore that if transient spirals are present, migration *must* take place. Figure 1 shows some examples of orbital histories from an idealized disk-formation simulation from Roškar et al. (2012). All of the stars were selected to be between 7-9 kpc at the end of the simulation, but clearly they originated in very different places in the disk and had diverse orbital histories. Interactions with spirals lead some stars to migrate inwards, others migrate outwards, some orbits become more eccentric and others circularize.

Investigating the reasons for migration in their models further, Roškar et al. (2012) confirmed that the most likely mechanism for the large migrations occurring on short timescales was the corotation resonance mechanism of Sellwood & Binney (2002). Figure 2 shows one piece of ev-

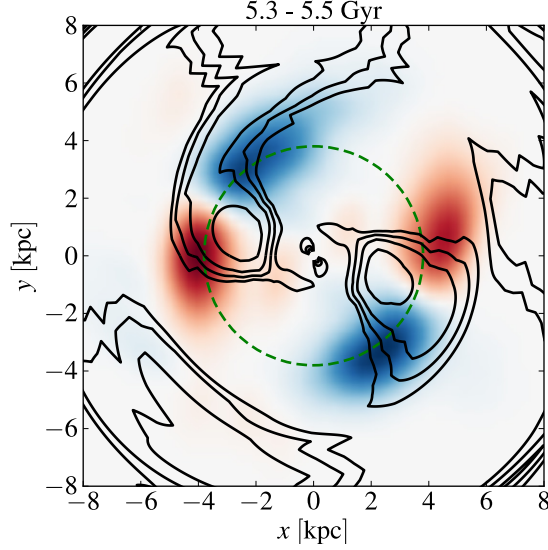


Figure 2: Inward (red) and outward (blue) migrator density in the time interval from 5.3-5.5 Gyr. The particles are chosen only by virtue of being in the top 5 percent of the ΔL_z distribution over this time interval. The contours show the stellar density reconstructed from the $m=2-4$ Fourier components. The dashed green line shows the location of the corotation resonance for circular orbits.

idence supporting this hypothesis: the top 5% of migrators in the specified time interval straddle the peak of the spiral density wave at the distance corresponding to its corotation resonance. In addition, Roškar et al. (2012) showed that the particles cross corotation precisely at the time of peak spiral amplitude and that their Jacobi integrals are conserved during the migration phase, all of which are expected in the corotation resonance picture (Sellwood & Binney, 2002).

In the models of Sellwood & Binney (2002) and Roškar et al. (2012), the spiral structure was shown, using Fourier methods (e.g. Sellwood & Athanassoula 1986), to host discrete pattern speeds with well-defined resonances. However, in some models the spiral structure pattern speed appears to match the rotation speed of the disk, essentially making the entire disk co-rotate with the spiral (e.g. Grand et al. 2012). Kawata et al. (2014) claim to have detected a signature of such spiral structure in the MW through a combination of measurements of gas and stellar motions. On the other hand, Meidt et al. (2009) used the modified Tremaine-Weinberg method to recover pattern speeds in several external galaxies and found that in general the pattern speeds were discrete, though the co-rotating spirals tend to be found in disks with lower surface-densities. The implications of co-rotating spirals is that mixing can be extremely efficient, though it is not clear whether it should lead to a unique chemo-dynamical signature.

Galactic disks are very prone to perturbations, so it is not surprising that other suggestions for redistributing material in the disks abound in the literature. For example, disks galaxies evolving in a cosmological environment are expected to undergo frequent encounters with substructure, which will not only disturb the disk (e.g. Kazantzidis et al. 2009) but also cause redistribution that can mimic migration due to the corotation resonance (Quillen et al., 2009).

Another important consideration is the influence of bars on the dynamical evolution of the

disk. Bars are robust structures that once formed evolve slowly. During the initial bar growth phase, angular momentum is redistributed very efficiently but the process also results in a significant amount of heating (Debattista et al., 2006). When fully formed, bars mostly heat the disk through the inner and outer Lindblad resonances. Since bars are not transient, efficient exchange of angular momentum is not expected at the corotation resonance, because stars are mostly trapped on horseshoe orbits. However, Minchev & Famaey (2010) argued that in the presence of overlapping resonances from other disk structure, the horseshoe orbits can be disrupted and stars may efficiently gain or lose angular momentum in a similar fashion to the transient spiral mechanism of Sellwood & Binney (2002). Such resonance overlap should result in non-linear evolution near the resonances whenever multiple patterns are present. Roškar et al. (2012) looked for signatures of this in their models where several spirals coexisted but found no clear evidence of this process. The orbits of stars undergoing rapid migration instead remained regular and clear signs of chaos were not found. Di Matteo et al. (2013) also argued that even steady bars can drive migration well after the instability epoch, but it is not clear whether the stellar dispersal in their model is due to heating and disk spreading or genuine radial migration. The diffusion of “hot stars” in barred galactic disks was shown already by Pfenniger & Friedli (1991), but those stars are distinctly chaotic and would result in non-circular orbits with large proper motions, which is very different from the Sellwood & Binney (2002) migrators described above.

In light of this discussion, it is crucial to point out that among the various mechanisms that can move material around the disk quickly and efficiently, migration due to the corotation resonance stands out because it causes very little radial heating. Satellite perturbations and bar formation, for example, affect the disk very differently and these diverse processes should not all be placed under the common label of “radial migration”. They describe very different physical processes and should be treated as such. However, just as the corotation migration causes little heating, it also has the largest effect on cool stellar populations. Therefore, models whose velocity dispersions are too high may not show signatures of radial mixing that are representative of the Sellwood & Binney (2002) migration mechanism. Collisionless models with strong bars are strong candidates for this category, since the bar formation episode heats the disk very rapidly and can serve to prevent more subtle disk structure needed for radial migration from forming.

4 Influence of migration on disk stellar populations

4.1 Solar Neighborhood and Outer Disks

One of the clear results of radial mixing is that it increases the diversity of stellar populations everywhere in the disk, which is especially important for Solar Neighborhood diagnostics like the AMR relation (Sellwood & Binney, 2002; Roškar et al., 2008a; Schönrich & Binney, 2009). The left panel of Figure 3 shows the distribution of formation radii for stars that end up between 7-9 kpc on mostly-circular orbits in the simulation of Roškar et al. (2008b), clearly indicating that stars born everywhere in the galaxy may find themselves in this part of the disk; 50% of these stars originated at $R < 6$ kpc. Such redistribution has obvious consequences for the AMR (middle panel) and the MDF (right panel) – the AMR is flattened and the dispersion in metallicity at each age increases significantly, while the MDF is significantly broadened.

A related and surprising early result of radial migration studies was that in a truncated disk (e.g. Pohlen & Trujillo, 2006), the stellar age profile is expected to show an upturn beyond the break radius (Roškar et al., 2008a). This prediction has been confirmed by a number of observational studies, both indirectly in terms of color gradients (Bakos et al., 2008) and directly from stellar population modeling using IFU observations (Yoachim et al., 2010) as well as HST resolved-star data (Radburn-Smith et al., 2012). While this prediction is not unique to radial

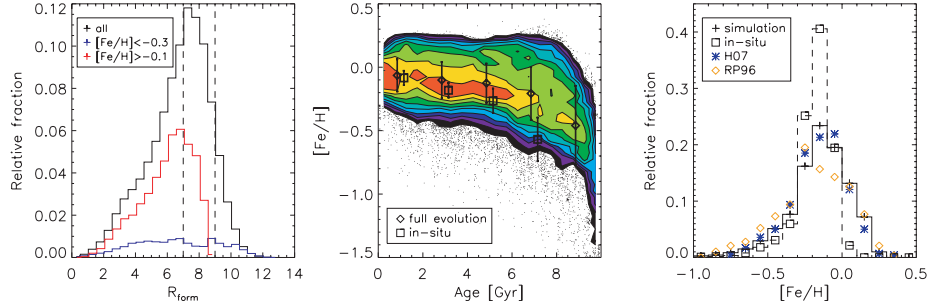


Figure 3: *Left*: distribution of formation radii for particles found within the solar neighborhood (defined to be $7 < R < 9$ kpc) at the end of the simulation from Roškar et al. (2008b). *Center*: The age-metallicity relation (AMR) in the solar neighborhood. Migration adds significantly to the dispersion in metallicities at each age. *Right*: Metallicity distribution function (MDF) is significantly broadened by migration.

mixing, since it is possible that the outer disks are influenced also by cosmological gas accretion and subsequent star formation (e.g. Sánchez-Blázquez et al., 2009), radial migration nevertheless can deposit significant amounts of material in outer disks. The redistributed material can perhaps overshadow the stars formed from accreted gas (Roškar et al., 2010) so the age inversion remains one of the few robust predictions of the process that have been confirmed by observations.

4.2 Migrated stars in the thick disk?

The thick disk is arguably one of the most coveted structures in the Milky Way, because of its potential to inform us about the early epoch of Milky Way’s disk formation. The possibility that radial migration can influence and partially populate the thick disk has therefore received much attention in recent years. The concept is quite simple: as a star changes its radial distance from the galactic center, the change in the vertical restoring force felt at different parts of the orbit affects its vertical displacement due to action conservation (Binney & Tremaine, 2008; Schönrich & Binney, 2012). This depends on how much of the potential is due to the disk (the MW disk is believed to be close to maximal, Bovy & Rix 2013) and the effect is compounded when the star’s guiding center is also changing.

In the model of Loebman et al. (2011) stars far away from the mid-plane at radii 7-9 kpc were found to be predominantly old and to originate in the inner disk, compared with stars closer to the mid-plane which were found to be younger and formed locally. They also showed that the V_ϕ - $[\text{Fe}/\text{H}]$ were anti-correlated for young stars and only slightly correlated for old stars in the solar neighborhood. This is easy to understand in the context of migration because young stars have not had time to migrate significantly and were therefore populating the local volume by virtue of mild heating. The young stars coming from the outer disk therefore had lower metallicities and higher rotational velocities, while the opposite was true for those coming from the inner disk. The old stars, on the other hand, had time to migrate and mix and therefore showed only a weak trend between metallicity and rotational velocity. The models qualitatively agree with recent observations such as from Lee et al. (2011), which also show a strong negative correlation between metallicity and V_ϕ for young (α -deficient) stars and a slight positive correlation for old (α -enhanced) stars. Quantitatively, the models do not agree with the data, though they also are not designed to reproduce the MW. The flattening of the V_ϕ - $[\text{Fe}/\text{H}]$ relation for old stars in the

model is certainly due to the radial mixing but in this particular model it does not turn into an overwhelmingly positive correlation so it is not clear whether an external process is required. Note that this observation poses a problem for all scenarios where the disk grows from the inside out because it requires the metal poor and metal rich stars to swap places in the disk at some well-defined epoch, when the α -enhancement is still high.

The influence of radial migration on the thickness of stellar populations in the models from Loebman et al. (2011) was explored in detail by Roškar et al. (2013). They showed that vertical thickening is a function of both age (i.e. heating) and radial displacement in the disk. In other words, orbits of stars of the same age, which experienced a similar amount of heating, become vertically more extended if they migrate outwards and more compressed if they migrate inwards. At the same time, stars from the same part of the disk that migrate by a similar amount are thicker if they are older. These results also showed that as stars move radially in the disk, their vertical velocity dispersions decreased for inward migration and increased for outward migration, confirming the effect shown by Minchev et al. (2012).

It should be stressed that the increase of scale-height with outward migration is not incompatible with the idea that the vertical action is conserved and a consequent decrease in velocity dispersion. If the velocity dispersion decreases as $\sigma_z \propto e^{-R/2R_d}$ (Minchev et al., 2012), and we make an assumption that the disk is in hydrostatic equilibrium with a sech^2 vertical density profile, it is easy to show that the scale height $h_z \propto e^{R/R_d}$ and therefore

$$\frac{h_{z,f}}{h_{z,i}} \propto \exp\left(\frac{\Delta R}{2R_d}\right), \quad (3)$$

where $h_{z,i}$ and $h_{z,f}$ are the initial and final scale heights of the population and ΔR is the change in radius. Thus, the change in scale height with radial displacement follows naturally from action conservation (see also Schönrich & Binney 2012 for a better approximation).

4.3 Limitations of simulations

The study of Roškar et al. (2013) showed unequivocally that a single population migrating outwards in their model will thicken, as expected based on the fact that the vertical actions are, on average, conserved (Solway et al., 2012). Does this mean that migration can form a thick disk? There are several caveats one must consider when interpreting simulated disks and their usefulness in generalizing disk thickening.

First, a disk must be resolved vertically in order for the simulation to say anything useful about the disk thickening process. In Roškar et al. (2013) the spatial resolution is 50 pc, but resolutions of > 100 pc are not uncommon in the literature dealing with vertical disk evolution. This means that within approximately one scale-height of the mid-plane, the forces on the particles are too low. The effect of this on migration studies has not been carefully scrutinized, but it is clear that it can affect how thick a migrated population looks compared to the stars already present at a given radius.

Second, when investigating the effect of migrators on the in-situ population, one must carefully consider the realism of the simulated star-forming layer, or in the case of collisionless simulations the assumptions about the initial disk structure. In Roškar et al. (2013), the gas disk out of which stars formed was significantly flared, meaning that although stars from the inner disk increased their thickness as they migrated outwards, they did not appear thicker than the in-situ population *because the in-situ population was thicker from the outset*. This is a general problem with hydrodynamic simulations using a temperature floor, since this means a fixed dispersion at all radii and consequently a radially increasing scale height h_z for the young stars because $h_z \propto \sigma_z/\Sigma$, where Σ is the disk surface density. However, while in that model the migrated stars

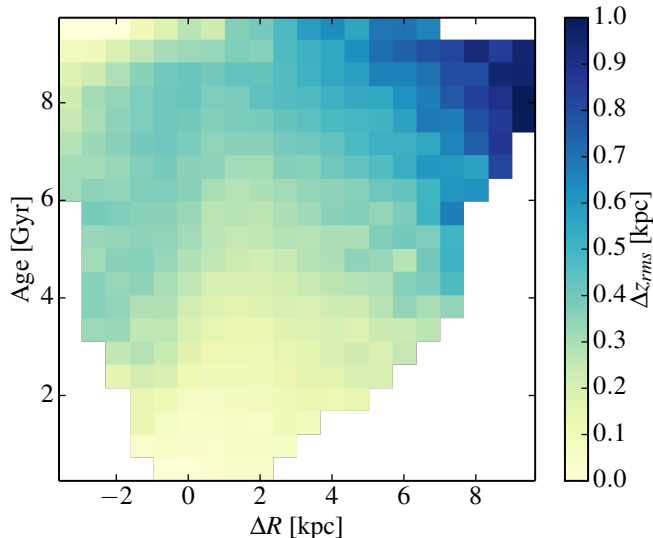


Figure 4: Total change in z_{rms} (proxy for thickness) for stars forming between 2-4 kpc as a function of age and change in radius.

therefore did not alone cause the thickening of the disk, they themselves still thickened as they migrated outwards. Therefore, they would not have been in the thicker part of the disk in the first place was it not for the migration. These stars therefore certainly influenced the chemical abundance distributions at large distances from the plane even if their effect on thickening was not large. The question of migration and the thick disk is not necessarily whether migration can *create* a thick disk, but whether the chemical abundance patterns that characterize the thick disk can be influenced by migration.

Finally, numerical relaxation effects may have a disproportionate influence on the vertical structure of disks (Sellwood, 2013). Therefore, numerical convergence tests must be performed to determine the robustness of simulated structures, since no obvious rule exists regarding sufficient particle numbers. This is further complicated by the fact that the evolution of disks is often highly stochastic, particularly when the disk supports multiple spiral modes (Sellwood & Debattista, 2009; Roškar et al., 2012). Nevertheless, such tests can reveal strong divergences for certain parameter choices: for example, Roškar et al. (2012) showed that if they used 500 pc softening instead of 100 pc or less, the disk formed completely different non-axisymmetric structure. Determining whether numerical relaxation is affecting the disk may be considerably more difficult for most applications.

5 Conclusions and challenges for the future

By enriching the stellar diversity in any given part of the Galactic disk, radial migration of stars complicates our interpretations of the stellar population record left behind by the process of disk formation. Migration due to the corotation resonance is inherently a complicated process, due to its dependence on resonances with transient structure that is difficult to constrain. Such

migration is special among the processes that redistribute material in galactic disks, because it introduces very little heating and therefore leaves behind no straightforward kinematic signatures. For this reason, N -body simulations have been instrumental in furthering our understanding of the underlying process.

At the same time, the simulations exploring the observational effects of migration have been very limited. On top of the dynamical complexity, simulations try to add in the processes of star formation and chemical enrichment in order to shed light on the expected effects in stellar population trends that migration leaves behind. While the purely dynamical modeling is potentially prone to numerical effects (as discussed above in relation to resolving disk scale heights), those are small compared to the uncertainties involved with sub-grid modeling required for capturing star formation and chemical enrichment within 3D N -body numerical simulations. We should therefore consider it a huge triumph that these simulations can agree with observations at all!

In the upcoming years, as data from Gaia, the GALAH survey (Freeman, 2012), and other similar large-scale projects become available, simulations will be critical in aiding our interpretations of those results. Nevertheless, to fully utilize those datasets, the theoretical efforts need to strive for an understanding of the leverage that various numerical and physical parameters exert on the process of radial mixing. On the other hand, one hopes that chemical tagging (Freeman & Bland-Hawthorn, 2002; Mitschang et al., 2013) will directly constrain how much radial mixing has taken place in the MW disk. Combining such constraints with a clearer picture of the underlying physical drivers of the mixing process, we should be in a much better position to unravel the past history of our MW disk.

Acknowledgments

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